Table 2 Comparison of experimental and analytical results for T300/5208[7] ($f=0.2$, hole diameter = 0.635 cm, $W/D=4$, $e/D=3$)

		Analytical				Experimental I[a]			Experimental II		
No.	Laminates	Ultimate load, N	Initial failure ply	Failure position, θ_f, deg	Failure mode	Av. failure load, N	Failure mode	Error, %	Av. failure load, N	Failure mode	Error, %
1	$(\pm 45)_s$	12290	−45	40	SO	11410	NT	− 7.7	12180	NT	− 1.0
2	$(0/\pm 45)_s$	14080	0	30	SO	13370	NT	− 5.3	15440	NT	8.0
3	$(0_2/\pm 45)_s$	23340	45	50	SO	17940	B	−30.1	19980	45°(NT)	−16.8
4	$(\pm 45_2/90)_s$	12030	45	30	SO	10600	NT	−13.5	11480	NT	− 4.8
5	$(\pm 45/90_2)_s$	11720	45	30	SO	9123	Nt	−28.5	9961	NT	−17.7

[a]For experimental results I, constraint torque of bolt = 0.56 N.m (5 lb.in.); for II, torque = 8.4 N.m (75 lb.in.).

Table 3 Comparison of experimental and analytical results for T300/5208[8] ($f=0.2$, hole diameter = 0.3175 cm, $W/D=8$, $e/D=6.5$)

		Analytical				Experimental[8 a]		Experimental[7]	
No.	Laminates	Stress, MPa	Initial failure ply	Failure position, θ_f, deg	Failure mode	Stress, MPa	Error, %	Stress, MPa	Error, %
1	$(0_2/\pm 45)_{2s}$	1096	45	50	SO	586.7	−86.8	842.6/938.4[a]	−30.1/−16.8
2	$(90_2/\pm 45)_{2s}$	554.2	45	30	SO	515.7	− 7.5	431.4/471.1	−28.5/−17.7
3	$(0/90/\pm 45)_{2s}$	876.5	0	20	B/SO	619.7	−41.4		

[a]See the footnote of Table 2 for the two different cases.

Experimental data obtained by Collings [11] and Wilkins[7] indicated that the failure load of a bolted joint with strong lateral constraint is 10-20% higher than the finger-tight case. A strong lateral constraint tends to limit the delamination possibility of a laminate, which is physically equivalent to the assumption that no consideration is given for delamination. This is confirmed by an overall small error percentage given under II and I in Table 2.

In a realistic structure, the direction of pin load may not coincide with one of the principal directions of a laminate. This can be handled without any difficulty by employing the results recently obtained by the authors[2] in connection with the same method adopted in this Note. In addition, considerations such as environmental effects and nonlinear and/or inelastic behavior would have to be included in the practical strength prediction of a bolted joint. The procedure and the results presented here can potentially reduce the number of tests that otherwise would be needed for the strength prediction of a mechanically fastened joint in laminated composites.

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Dynamic Response of Orthotropic, Homogeneous, and Laminated Cylindrical Shells

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Introduction

RECENT developments in the analysis of plates and shells laminated of fiber-reinforced materials indicate that thickness has more pronounced effects on the behavior of composite laminated shells than on isotropic laminated shells. Also, due to low transverse shear moduli relative to in-plane Young's moduli, transverse shear deformation effects are even more pronounced in composite laminates. Reliable prediction of the response characteristics of high modulus composites therefore requires the use of shear deformation theories.

Several theories to analyze laminated shells have been put forward in which the effects of transverse shear deformation are taken into account. The first formulation of such a theory for orthotropic laminated cylindrical shells appears to be due to Dong and Tso.[2] Similar works on laminated shells can be found in Refs. 3-5. An exposition of various shell theories is

Received June 10, 1983; revision received Nov. 27, 1984.

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available in Ref. 6. Many different higher order shell theories have been proposed that take into account the effects of transverse components of strain. Typical examples of such theories are cited in Refs. 7-10. A higher order theory for dynamic response of cylindrical shell is developed herein for application to orthotropic homogeneous and laminated shells. This Note is an extension of the work developed in Ref. 11 for application to isotropic homogeneous shells. The frequencies obtained from the present analysis are used to evaluate other shell theories.

Derivation of Governing Equations of Motion

Let us select a cylindrical system such that the x-θ plane coincides with the midplane of the shell of mean radius a, length L, and total thickness h (h_1,h_2,h_3 are thicknesses of individual layers in the case of layered shell). The components of displacements are assumed as follows

$$u(x,\theta,z,t) = u_0(x,\theta,t) + \xi u_1(x,\theta,t) - z w_{0,x}$$

$$v(x,\theta,z,t) = \left(\frac{a+z}{a}\right)\left[v_0(x,\theta,t) + \xi v_1(x,\theta,t) - \frac{z}{a+z} w_{0,\theta}\right]$$

$$w(x,\theta,z,t) = w_0(x,\theta,t) \tag{1}$$

where $\xi = z[1-(4z^2/3h^2)]$; the comma indicates the differentiation with respect to the letter(s) followed by it; u,v,w are the displacements in x,θ,z directions, respectively; and t is the time coordinate. It may be noted that the preceding form of expressions for displacement components results in the parabolic variation of transverse shear strains and further discussion on the selection of these forms may be found in Ref. 11. We have the following definitions for stress resultants appropriate to the present method of analysis,

$$\begin{bmatrix} N_x & N_\theta \\ M'_x & M'_\theta \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} 1 \\ \xi \end{bmatrix} (\sigma_x, \sigma_\theta)\left(\frac{a+z}{a}\right) dz$$

$$(M_\theta, M_{\theta x}) = -\int_{-h/2}^{h/2} (\sigma_\theta, \tau_{x\theta}) z\, dz$$

$$\begin{bmatrix} N_{\theta x} & M'_{\theta x} \\ N_{x\theta} & M'_{x\theta} \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} 1 \\ [(a+z)/a]^2 \end{bmatrix} (1,\xi)\tau_{x\theta}\, dz$$

$$(M_x, M_{x\theta}) = -\int_{-h/2}^{h/2} (\sigma_x, \tau_{x\theta}) z \left(\frac{a+z}{a}\right) dz \tag{2}$$

Also,

$$Q_x = M_{x,x} + (1/a)(M_{\theta x} + M_{x\theta})_{,\theta} + F_1 u_{0,tt} + \Gamma_2 u_{1,tt} - F_2 w_{0,xtt}$$

$$Q_\theta = (1/a) M_{\theta,\theta} + (M_{\theta x} + M_{x\theta})_{,x} + \left(F_1 + \frac{1}{a} F_2\right)\frac{1}{a} v_{0,tt} + \left(\Gamma_2 + \frac{1}{a}\Gamma_3\right)\frac{1}{a} v_{1,tt} - \frac{1}{a^2} F_2 w_{0,\theta tt}$$

Using Hamilton's principle, the following equations of dynamic equilibrium can be obtained.

$$[L]\{\delta\} = \{q\} \tag{3}$$

where, $p = (q_{z0} - q_{zi}) + (h/2a)(q_{z0} + q_{zi})$; q_{z0}, q_{zi} are the applied forces in the z direction on outer and inner surfaces of the shell, respectively; and

$$\delta^T = [u_0\ v_0\ u_1\ v_1\ w_0]$$

$$q^T = [0\ 0\ 0\ 0\ p]$$

The operators L_{ij} are symmetric and have the following forms:

$$\begin{bmatrix} L_{11} \\ L_{13} \\ L_{22} \\ L_{24} \end{bmatrix} = \begin{bmatrix} A^0_{11} \\ \bar{\alpha}^1_{11} \\ \bar{\alpha}^0_{66} \\ \bar{\alpha}^1_{66} - (4/3h^2)\alpha^3_{66} \end{bmatrix} (\ )_{,xx} + \begin{bmatrix} a^2 D^0_{66} \\ a^2\eta^1_{66} \\ A^0_{22} \\ \alpha^1_{22} \end{bmatrix} \frac{1}{a^2}(\ )_{,\theta\theta} + \begin{bmatrix} F_0 \\ \Gamma_1 \\ \bar{\Gamma}_0 \\ \bar{\Gamma}_1 - (4/3h^2)\bar{\Gamma}_3 \end{bmatrix} (\ )_{,tt}$$

$$\begin{bmatrix} L_{12} \\ L_{14} \\ L_{23} \\ L_{34} \end{bmatrix} = \begin{bmatrix} A^0_{12} + A^0_{66} \\ \alpha^1_{12} + \alpha^1_{66} \\ \alpha^1_{12} + \alpha^1_{66} \\ \bar{\bar{\alpha}}^2_{12} + \bar{\bar{\alpha}}^2_{66} \end{bmatrix} \frac{1}{a}(\ )_{,x\theta}$$

$$\begin{bmatrix} L_{15} \\ L_{35} \end{bmatrix} = -\begin{bmatrix} A^1_{11} \\ \alpha^2_{11} \end{bmatrix}(\ )_{,xxx} - \begin{bmatrix} B^1_{12} + 2a^2 D^1_{66} + aD^2_{66} \\ \beta^2_{12} + 2a^2\eta^2_{66} + a\eta^3_{66} \end{bmatrix} \times \frac{1}{a^2}(\ )_{,x\theta\theta} + \begin{bmatrix} B^0_{12} \\ \beta^1_{12} \end{bmatrix}\frac{1}{a}(\ )_{,x} - \begin{bmatrix} F_1 \\ \Gamma_2 \end{bmatrix}(\ )_{,xtt}$$

$$\begin{bmatrix} L_{25} \\ L_{45} \end{bmatrix} = -\begin{bmatrix} A^1_{12} + 2A^1_{66} + (1/a)A^2_{66} \\ \alpha^2_{12} + 2\alpha^2_{66} + (1/a)\alpha^3_{66} \end{bmatrix}\frac{1}{a}(\ )_{,xx\theta} - \begin{bmatrix} B^1_{22} \\ \beta^2_{22} \end{bmatrix}\frac{1}{a^3}(\ )_{,\theta\theta\theta} + \begin{bmatrix} B^0_{22} \\ \beta^1_{22} \end{bmatrix}\frac{1}{a}(\ )_{,\theta} - \begin{bmatrix} F_1 + (1/a)F_2 \\ \Gamma_2 + (1/a)\Gamma_3 \end{bmatrix}\frac{1}{a}(\ )_{,\theta tt}$$

$$\begin{bmatrix} L_{33} \\ L_{44} \end{bmatrix} = \begin{bmatrix} \bar{\bar{\alpha}}^2_{11} \\ \bar{\alpha}^2_{66} - (8/3h^2)\bar{\alpha}^4_{66} + (16/9h^4)\bar{\alpha}^6_{66} \end{bmatrix}(\ )_{,xx} + \begin{bmatrix} \bar{\bar{\eta}}^2_{66} \\ \bar{\bar{\alpha}}^2_{22} \end{bmatrix}\frac{1}{a^2}(\ )_{,\theta\theta} + \begin{bmatrix} \bar{\bar{\Gamma}}_2 \\ \bar{\Gamma}_2 - (8/3h^2)\bar{\Gamma}_4 + (16/9h^4)\bar{\Gamma}_6 \end{bmatrix} \times (\ )_{,tt} - \begin{bmatrix} A^0_{44} - (8/h^2)A^2_{44} + (16/h^4)A^4_{44} \\ \bar{\alpha}^0_{55} - (8/h^2)\bar{\alpha}^2_{55} + (16/h^4)\bar{\alpha}^4_{55} \end{bmatrix}$$

$$L_{55} = A^2_{11}(\ )_{,xxxx} + (2B^2_{12} + 3a^2 D^2_{66} + a^2\bar{\eta}^2_{66})\frac{1}{a^2}(\ )_{,xx\theta\theta} + \frac{1}{a^2}D^2_{22}(\ )_{,\theta\theta\theta\theta} - \frac{2}{a}B^1_{12}(\ )_{,xx} - \frac{2}{a}D^1_{22}(\ )_{,\theta\theta} + D^0_{22}(\ ) + F_2(\ )_{,xxtt} + \frac{1}{a^2}F_2(\ )_{,\theta\theta tt} - F_0(\ )_{,tt}$$

where $(\ )_{,xx}$, etc., refers to derivatives of the column matrix of generalized displacements in Eqs. (3) and

$$A_{ij}^k = \int_{-h/2}^{h/2} C_{ij} z^k \left(\frac{a+z}{a}\right) dz \qquad B_{ij}^k = \int_{-h/2}^{h/2} C_{ij} z^k dz$$

$$D_{ij}^k = \int_{-h/2}^{h/2} C_{ij} z^k \frac{1}{a(a+z)} dz \qquad F_k = \int_{-h/2}^{h/2} \rho z^k \left(\frac{a+z}{a}\right) dz$$

$$\begin{bmatrix} \alpha_{ij}^k \\ \beta_{ij}^k \\ \eta_{ij}^k \\ \Gamma_k \end{bmatrix} = \begin{bmatrix} A_{ij}^k & A_{ij}^{k+2} \\ B_{ij}^k & B_{ij}^{k+2} \\ D_{ij}^k & D_{ij}^{k+2} \\ F_k & F_{k+2} \end{bmatrix} \begin{bmatrix} 1 \\ -4/3h^2 \end{bmatrix}$$

$$\begin{bmatrix} \bar{\alpha}_{ij}^k \\ \bar{\beta}_{ij}^k \\ \bar{\eta}_{ij}^k \\ \bar{\Gamma}_k \end{bmatrix} = \begin{bmatrix} A_{ij}^k & A_{ij}^{k+1} & A_{ij}^{k+2} \\ B_{ij}^k & B_{ij}^{k+1} & B_{ij}^{k+2} \\ D_{ij}^k & D_{ij}^{k+1} & D_{ij}^{k+2} \\ F_k & F_{k+1} & F_{k+2} \end{bmatrix} \begin{bmatrix} 1 \\ 2/a \\ 1/a^2 \end{bmatrix}$$

$$\begin{bmatrix} \bar{\bar{\alpha}}_{ij}^k \\ \bar{\bar{\beta}}_{ij}^k \\ \bar{\bar{\eta}}_{ij}^k \\ \bar{\bar{\Gamma}}_k \end{bmatrix} = \begin{bmatrix} A_{ij}^k & A_{ij}^{k+2} & A_{ij}^{k+4} \\ B_{ij}^k & B_{ij}^{k+2} & B_{ij}^{k+4} \\ D_{ij}^k & D_{ij}^{k+2} & D_{ij}^{k+4} \\ F_k & F_{k+2} & F_{k+4} \end{bmatrix} \begin{bmatrix} 1 \\ -8/3h^2 \\ 16/9h^2 \end{bmatrix}$$

Finally, the boundary conditions along the edge of the shell require that one member of each of the following six pairs must be prescribed.

$$N_x u_0 \ N_{x\theta} v_0 \ M_x' u_1 \ M_{x\theta}' v_1 \ Q_x w_0 \ M_x w_{0'x} \quad (\text{along } x = \text{const})$$

$$N_{\theta x} u_0 \ N_\theta v_0 \ M_{\theta x}' u_1 \ M_\theta' v_1 \ Q_\theta w_0 \ M_\theta w_{0'\theta} \quad (\text{along } \theta = \text{const})$$

It may be observed that thin shell theory equations can be obtained from Eq. (3) by deleting the terms corresponding to u_1 and v_1 from L_{ij} matrix.

Results and Discussions

The free vibration of simply-supported shell with no axial constraint is analyzed using equations of motion [Eq. (3)]. The solution of Eq. (3) is assumed in the following modal form,

$$(u_0, u_1) = (A_1, A_3)\Phi_1(x,\theta,t)$$

$$(v_0, v_1) = (A_2, A_4)\Phi_2(x,\theta,t); \quad w_0 = A_5 \Phi_3(x,\theta,t) \tag{4}$$

where

$$\Phi_1(x,\theta,t) = \cos(m\pi x/L)\cos n\theta \sin\Omega t$$

$$\Phi_2(x,\theta,t) = \sin(m\pi x/L)\sin n\theta \sin\Omega t$$

$$\Phi_3(x,\theta,t) = \sin(m\pi x/L)\cos n\theta \sin\Omega t$$

Ω is the angular frequency; and m,n are axial and circumferential mode numbers, respectively. Substituting Eq. (4) in Eq. (3), in the absence of applied forces, the problem can be reduced to finding the eigenvalues and the eigenvectors of the cylinder, for a given set of (m,n) values.

The results (frequencies) are presented in the tabular form for different geometric and material parameters considered, along with the other shell theories. The following geometric parameters have been taken: $\lambda = 0.5\pi, \pi, 2\pi, 4\pi$; $h/a = 0.05, 0.1, 0.15, 0.2$; $\zeta_1 = \zeta_3 = \zeta = 1, 5, 10, 15$; where $\lambda = m\pi a/L$, $\zeta_1 = h_2/h_1$, and $\zeta_3 = h_2/h_3$. The orthotropic material properties considered in the examples have been taken from Ref. 12. In the case of layered shells, the mass densities for all plies have been taken as constant ($\rho_1 = \rho_2 = \rho_3 = \rho$). The values of the shear correction factors used in calculating the numerical results for the shear deformation theory have been taken as $\pi^2/12$.

Orthotropic Homogeneous Shells

From the results obtained for homogeneous shells (results are not given for the sake brevity), it has been observed that the agreement between the shear deformation theory (SDT) and the present analysis is excellent, particularly for lower values of λ. The frequency values predicted by Flügge shell theory (FST) are in good agreement for $\lambda \le 0.5\pi$, and they differ considerably for higher values of λ (the maximum difference is about 52% for $\lambda = 4\pi$). The frequency values, other than those corresponding to the flexural mode of vibration, predicted by SDT and FST compare very well with the present analysis. Hence, in subsequent discussions on homogeneous and layered composite shells only the lowest natural frequency values have been considered.

It has also been seen that the difference in the frequency values predicted by SDT and FST increases as the shell thickness increases. The maximum difference in SDT is about −3.3% for $h/a = 0.2$, and in FST it is about 113% for

Table 1 Comparison of lowest natural frequency values Ω/Ω_0 for two-ply (0/90) laminated shell for different values of $\zeta\ (=h_2/h_1)$ and $\lambda = 4\pi$, $n = 2$[a]

h/a		1	5	10	15
	P	3.127435	3.177554	3.122052	3.068650
0.05	S	3.117434	3.169294	3.111273	3.058520
	F	3.330101	3.362923	3.279304	3.208832
	P	4.747570	4.645713	4.560325	4.481185
0.10	S	4.693478	4.600565	4.501343	4.423619
	F	5.880767	5.707303	5.512817	5.360394
	P	5.737212	5.590045	5.527269	5.456590
0.15	S	5.613727	5.493074	5.403325	5.333200
	F	7.992275	7.728920	7.525059	7.337676
	P	6.342699	6.156039	6.116822	6.060382
0.20	S	6.131890	5.999250	5.923390	5.865487
	F	9.689085	9.303546	9.083850	8.880311

[a]P, present; S, shear deformation[1]; F, Flügge theory[13]: $\Omega_0^2 = E_x/\rho a^2$, $E_x = 1$, $E_\theta = 0.245614$, $\mu_{x\theta} = 0.277$, $G_{\theta z} = 0.0877192$, $G_{x\theta} = G_{xz} = 0.1$.

Table 2 Comparison of lowest natural frequency values Ω/Ω_0 for three-ply (0/90/0) laminated shell for different values of $\zeta\ (=h_2/h_1 = h_2/h_3)$ and $\lambda = 4\pi$, $n = 2$[a]

h/a		1	5	10	15
	P	4.057502	3.807452	3.554208	3.399059
0.05	S	4.058200	3.779384	3.524447	3.373638
	F	4.709607	4.221290	3.834631	3.620862
	P	5.694970	5.403872	5.122172	4.924206
0.10	S	5.676497	5.288363	4.985559	4.797995
	F	8.112165	7.564961	6.745997	6.288352
	P	6.453898	6.259959	6.067436	5.906379
0.15	S	6.378206	6.055216	5.817730	5.666325
	F	12.129970	10.559861	9.399144	8.759593
	P	6.868500	6.716040	6.595593	6.473787
0.20	S	6.702116	6.428202	6.244791	6.127860
	F	14.853824	12.918700	11.492265	10.701562

[a]P, present; S, shear deformation; F, Flügge theory: $\Omega_0^2 = E_x/\rho a^2$, $E_x = 1$, $E_\theta = 0.245614$, $\mu_{x\theta} = 0.277$, $G_{\theta z} = 0.0877192$, $G_{x\theta} = G_{xz} = 0.1$.

$h/a = 0.2$, $n = 2$, and $\lambda = 4\pi$. Also, it has been observed that the difference in frequency values predicted by SDT and FST increases compared to the present analysis as E_x/E_θ, E_x/G_{xz}, and $E_x/G_{\theta z}$ values of the shell material increases. In general, SDT underestimates the frequency values and FST overestimated the same as compared to the present analysis.

Layered Composite Shells

Tables 1 and 2 show the nondimensionalized lowest natural frequency values for two- and three-layered (cross-ply) shells. The results were also obtained for 90/0 (two-layered) and 90/0/90 (three-layered) arrangement of plies and are not given here for brevity. However, observations on those results are included in the discussions to follow. In general, it may be seen that SDT underestimates the frequency values and FST overestimates the same as compared to the present analysis.

From Table 1, for two-layered shells, it may be observed that for $n = 2$ and $\lambda = 4\pi$, the agreement between SDT and the present analysis is very good for lower values of h/a and SDT differs by about -3.3% for $h/a = 0.2$ and 0/90 arrangement. The agreement in the frequency values predicted by SDT is better in the case of 90/0 arrangement as compared to the corresponding shell of 0/90 arrangement. The difference between FST and the present analysis is very much higher.

There is little change in the frequency values as ζ value increases for a given h/a value. As ζ value increases the frequency values decrease for 0/90 arrangement, whereas they increase for 90/0 stacking arrangement for the shell.

From Table 2, for three-layered shells, it may be seen that for $n = 2$ and $\lambda = 4\pi$, the agreement between SDT and the present analysis is good for lower h/a values (difference is $< 1\%$ for $h/a \leq 0.05$). As h/a value increases SDT differs from the present analysis considerably (about 5.5% for $h/a = 0.2$, $\zeta = 15$ and 0/90/0 shell). The agreement in the frequency values predicted by SDT is better in the case of 90/0/90 shell as compared to the corresponding 0/90/0 shell. As mentioned earlier, the difference between FST and the present analysis is very much higher.

As said earlier, for two-layered shells, as ζ value increases frequency values decrease for 0/90/0 arrangement, whereas they increase for 90/0/90 arrangement, for a given h/a value. For two- and three-layered shells for various λ and n values considered, it was observed that the percent difference in SDT and FST values, when compared to the present analysis, is greater for higher values of λ and n.

Conclusions

A higher order theory for dynamic response of orthotropic homogeneous and composite laminated cylindrical shells is developed. The theory accounts for in-plane inertia, rotary inertia and shear deformation effects. The proposed method assumes parabolic variations across the thickness of shell for transverse shear strains, with zero values at the extreme fibres of the shell. From the results obtained it may be concluded that the shear deformation theory differs appreciably from the present analysis in the following cases: 1) orthotropic homogeneous shells with higher values of E_x/E_θ, E_x/G_{xz}, and $E_x/G_{\theta z}$; 2) homogeneous and laminated shells with short axial wavelengths and higher thickness to radius ratios; and 3) homogeneous and laminated shells with higher circumferential mode number.

Thus, for the better prediction of the response characteristics, where the above mentioned cases are involved, the higher order theories like the one presented here may be used.

Acknowledgments

The author thanks Professor L. K. Stevens for his guidance during the course of this work. It is also a pleasure to acknowledge the constructive comments of the reviewers on an earlier version of the manuscript.

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